PEVA

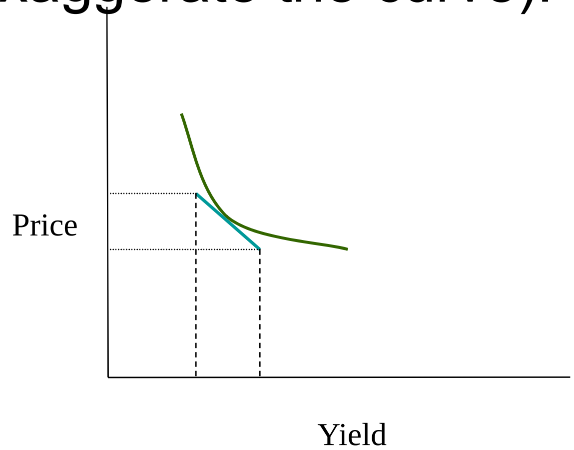
Day 5 Slides

Fixed-Income Attribution

A Review of Duration and Convexity Principles

- To Compute Duration:
  - First, write the bond pricing equation
  - Second, take the derivative with respect to yield: this is the approximate dollar price change for a small change in yield!
  - Third, collect terms
  - Fourth, divide both sides by P: this is the approximate %-age price change for a small change in yield!
  - This is the (negative of) modified duration!

To Illustrate, Look at the 9% 25-Year Bond (we'll exaggerate the curve):



$$P = \frac{C}{\left(1+\frac{y}{2}\right)} + \frac{C}{\left(1+\frac{y}{2}\right)^2} + \frac{C}{\left(1+\frac{y}{2}\right)^3} + \ldots + \frac{C+M}{\left(1+\frac{y}{2}\right)^n}$$

$$\frac{dP}{d\left(\frac{y}{2}\right)} = \frac{-C}{\left(1+\frac{y}{2}\right)^2} + \frac{-2C}{\left(1+\frac{y}{2}\right)^3} + \frac{-3C}{\left(1+\frac{y}{2}\right)^4} + \ldots + \frac{-n(C+M)}{\left(1+\frac{y}{2}\right)^{n+1}}$$

$$= -\frac{1}{\left(1+\frac{y}{2}\right)} \left[\frac{1C}{\left(1+\frac{y}{2}\right)} + \frac{2C}{\left(1+\frac{y}{2}\right)^2} + \frac{3C}{\left(1+\frac{y}{2}\right)^3} + \ldots + \frac{n(C+M)}{\left(1+\frac{y}{2}\right)^n} \right]$$

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$$\frac{dP}{d\left(\frac{y}{2}\right)} \bullet \left[\frac{1}{P}\right] = -\frac{1}{\left(1+\frac{y}{2}\right)} \underbrace{\left[\frac{1C}{\left(1+\frac{y}{2}\right)} + \frac{2C}{\left(1+\frac{y}{2}\right)^2} + \frac{3C}{\left(1+\frac{y}{2}\right)^3} + \ldots + \frac{n(C+M)}{\left(1+\frac{y}{2}\right)^n} \right] \bullet \left[\frac{1}{P}\right]}_{\text{“Macaulay Duration”}}$$

“Macaulay Duration”

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- We end up with:

$$\frac{dP}{d\left(\frac{YTM}{2}\right)} \times \left[\frac{1}{P}\right] = -\frac{1}{\left(1+\frac{YTM}{2}\right)} \times (\mathit{Macaulay\ Duration})$$

$$= -\ (\mathit{Modified\ Duration})$$

What is modified duration?

- It is the (negative) of:
  - "the approximate %-age change in price (rate-of-return) for a small change in yield"

Durations are quoted in YEARS!

- So, the last step is to divide by the number of periods per year (usually 2 in bond markets):

Duration in years =

(Duration in periods) / (# periods per year)

Example:

Bond	**Macaulay Duration (years)**	**Modified Duration (years)**
9%/5-year	4.13	3.96
9%/25-year	10.33	9.88
6%/5-year	4.35	4.16
6%/25-year	11.10	10.62
0%/5-year	5.00	4.78
0%/25-year	25.00	23.92

Properties of Duration

- The duration of any coupon bond is less than its maturity (both modified and Macaulay duration)
- Macaulay duration of a zero equals its maturity
- Modified duration of a zero < maturity
- Lower coupon bonds (all else held constant) have longer duration (both modified and Macaulay)

- Finally, the lower the yield (with constant coupon and maturity), the higher the duration:

A 9% coupon/25-year bond:

Yield (%/yr)	Modified Duration (years)
7	11.21
8	10.53
11	8.70
13	7.66
14	7.21

- Bottom line: the factors that increase bond price volatility also increase duration!
  - Therefore, the greater the duration, the greater the bond price volatility

Uses of Modified Duration

- Approximating the %-age price change (rate-of-return) due to a change in yield:

$$\frac{dP}{d\left(\frac{YTM}{2}\right)} \times \left[\frac{1}{P}\right] = -(\,Modified\ Duration)$$

$$\frac{dP}{P} = -(\,Modified\ Duration) \times d\left(\frac{YTM}{2}\right)$$

Example: A 25-yr, 6% bond yielding 9%

- Modified duration = 10.62 years
- What is the approximate %-age price change with a 10 bp change in yield (i.e., from 9.0 to 9.1%/yr)?
- Solution:

$$\frac{dP}{P} = -(10.62) \times (.0010) = -.01062 = -1.062\ \%$$

How good is this approximation?

- Actual price at a yield of 9.1 is -1.05 % (you have to actually calculate the price at 9.1 to figure this out)
- So, the estimate (-1.06 %) is close to the actual (-1.05 %)
- The estimated %-age price change using duration is close when yield shifts are small

Try a change of 200 bp:

- Modified duration = 10.62 years
- What is the approximate %-age price change with a 200 bp change in yield (i.e., from 9.0 to 11.0 %/yr)?
- Solution:

$$\frac{dP}{P} = -(10.62) \times (.0200) = -.2124 = -21.24\ \%$$

- However, actual change is -18.03 %, compared to our estimate of -21.24%
- So, using duration to estimate %-age price change causes large errors with large yield shifts
- Note: Using duration to estimate the new price always underestimates the new price!

- Problem: Duration always underestimates the new price after a big yield change
- The problem is worse for bonds having a lot of convexity
- Solution: Include a convexity correction factor!

Convexity

$$Convexity = \frac{d^2 P}{d\left(\frac{YTM}{2}\right)^2} \times \frac{1}{P}$$

- You can think of this as the change in the modified duration with change in yield

Convexity Properties

- **P-1:** As the yield of a given bond increases, the convexity decreases
- **P-2:** Lower coupon bonds (with same maturity and yield) have more convexity
- **P-3:** Longer-maturity bonds (with same coupon rate and yield) have more convexity

Last point: traders are willing to "pay for convexity," since more convex bonds have a higher price whether yields rise or fall!

Yield Curve Basics

- The Treasury yield curve is often modeled as moving with:
  - Parallel shifts, plus
  - Twists, plus
  - Butterfly effects
- (Let's look at dynamics of yield curve using Craig Holden's spreadsheet)
- (See, also, page 52 of Colin "Fixed-Income Attribution")

- These 3 effects are somewhat ambiguous:
  - Example: Colin, Figure 6.2 (pages 53-54)
  - Example: Twist at short end plus twist at long end of curve can equal parallel shift
- Must fix a twist maturity point to solve this problem
  - Should be chosen according to the duration of each bond in the fixed-income portfolio (or, duration of manager's mandate)
  - Otherwise, attribution analysis will be misleading

Polynomial Modeling of Yield Curve Dynamics

- Let y(m) = yield as a function of maturity
- S = twist point

$$y(m) = a_0 + a_1(m - S) + a_2(m - S)^2$$

- Then,

$$\delta y_{shift} = a_0^{t+1} - a_0^t$$

$$\delta y_{twist} = \left(a_1^{t+1} - a_1^t\right) \bullet (m - S)$$

$$\delta y_{butterfly} = \left(a_2^{t+1} - a_2^t\right) \bullet (m - S)^2$$

Approximating Fixed-Income Returns Using Yield, Duration and Convexity

- Let r = return on bond
- y = ytm
- MD = modified duration
- C = convexity

$$r = \underbrace{y \cdot \delta t}_{\text{simple yield return}} - \underbrace{MD \cdot \delta y}_{\text{parallel shift}} + \underbrace{\frac{1}{2} C\left(\delta y\right)^2}_{\text{convexity correction}}$$

Yield Return Decomposition

$$r_{yield} = r_{coupon} + r_{rolldown}$$

$$r_{coupon} = c \cdot \delta t$$

- Simple yield may be further decomposed into that of an equivalent maturity government bond + a yield spread

Treasury Curve Shift Decomposition

$$r_{\substack{parallel\\shift}} = -MD \cdot \delta y$$

- This duration yield may be further decomposed into the duration yield of the benchmark portfolio + the difference in duration yield of the portfolio (minus the benchmark)—the "duration bet"—both computed using shifts in gov't yield curve

- Remaining return of the equivalent gov't bond portfolio is called "yield curve repositioning effect"
  - Captures twist and butterfly effects on gov't yield curve

Roll Return

- Freeze the gov't yield curve
- "Roll return" is the return derived from the shortening of the maturity of the bond portfolio (upward-sloping yield curve implies positive roll return)

Other Components

- Credit return
- Security selection return
- Optionality return

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Survivorship and Selection Biases

Effects of Survivor Bias

- Impacts estimate of performance of the "average fund"
- Impacts estimate of the persistence in performance
- Impacts estimate of the relation between performance and fund characteristics, e.g., fund size or expenses

Bias in Average Fund Performance

- Length of survival period affects the bias in average fund alphas
  - For example, a database that contains only funds that survive over a 10-year period has greater bias than one that contains only funds that survive over a 5-year period
- Survival rule of the market impacts the bias
  - If funds die after one year of bad performance, this will impose a bigger survival bias for average fund performance than if funds die only after a 10-year poor performance period

Bias in Performance Persistence

- Survival rule impacts the bias
  - If funds die after short periods of poor performance, then only persistent winning funds live, making the bias in persistence stronger
    - This results in higher persistence in the survivor-biased dataset
  - If funds die after long periods of poor performance, then only persistent losing funds die.
    - This results in lower persistence in the survivor-biased dataset

Bias in the Relation Between Performance and Fund Characteristics

- The bias will only occur for a fund characteristic that is correlated with survival
  - For instance, fund size, since small funds are more likely to die after poor performance
  - This will make small funds appear to outperform large funds in the survivor-biased dataset

Other Biases…

- Backfilling performance records
  – Incubator funds (Evans, 2010, JF)
- Merged fund performance histories
  – Funds may choose to use more successful fund's history
- Hedge fund database inclusion biases
  – Liquidated funds missing
  – Super-successful funds missing

Bootstrapping Alphas

Kosowski, Timmermann, Wermers, and White (Journal of Finance, December 2006)

Trying to Understand Hedge Fund Risks…

No Academic Studies Explicitly Model the Role of "Luck" in Performance

- Important to model the distribution of performance measures
  - Theory: difficult to derive the distribution in theory, as it depends on:
    - Assumptions about timing vs. selectivity abilities of the fund manager
    - Assumptions about structure of factor-mimicking portfolio returns (e.g., the book-to-market "factor")
    - Assumptions about the "tournaments" that might be played by managers as the performance "game" proceeds through time

Our Paper Bootstraps the Distribution of Performance Measures for the Funds

- Since there is little agreement on the correct model to use for “risk-adjustment” or “style-adjustment,” we bootstrap the distribution of many widely used models
- Our main objective in this paper is to show that bootstrapped p-values can substantially change inferences about managerial abilities for several different models

Unconditional Performance Models (Regressors are in Parentheses)

- Pure selectivity models
  - – Jensen Measure (RMRF)
  - – Fama-French Measure (RMRF, SMB, HML)
  - – Carhart Measure (RMRF, SMB, HML, PR1YR)
- Models of Timing and Selectivity
  - – Henriksson and Merton (RMRF+ = max(0,RMRF))
  - – Treynor and Mazuy (RMRF, $RMRF^2$)

Conditional β Models

- Conditional Jensen (RMRF, z_1*RMRF, z_2*RMRF, z_3*RMRF, z_4*RMRF, z_5*RMRF)
- Conditional F-F (RMRF, z_1*RMRF z_1*SMB, z_1*HML, etc.)
- Conditional Carhart
- Conditional Treynor-Mazuy
- Conditional Henriksson-Merton

Conditional α and β Models

- Conditional Jensen (z_1, z_2, z_3, z_4, z_5, RMRF, z_1*RMRF, z_2*RMRF, z_3*RMRF, z_4*RMRF, z_5*RMRF)
- Conditional F-F (z_1, z_2, z_3, z_4, z_5,RMRF, z_1*RMRF z_1*SMB, z_1*HML, etc.)
- Conditional Carhart
- Conditional Treynor-Mazuy
- Conditional Henriksson-Merton

The Hypothesis Test for "Star" Fund Managers

- H_0 : max α_i <= 0 (i = 1,…,L)
- H_1 : max α_i > 0 (i = 1,…,L)
- The distribution function for these maximum α-statistics is bootstrapped
- Analogous tests for "goat" fund managers

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The Bootstrapping Procedure

- Step 1: Obtain "alpha" estimates, factor loadings, and residuals from the actual fund returns and factor portfolios
- Example: With the Carhart model,
  - A. Regress:

$$R_i - R_f = \alpha + \beta * RMRF + s * SMB + h * HML + p * PR1YR + \varepsilon$$

  - B. Save estimated time-series residuals and regression coefficients, as well as estimated "alpha"

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- Step 2: Resample a full sequence of residuals, $\{\varepsilon_t\}$, one for each time-period, for fund *i* for b=1,…,B (draw "B" bootstrapped residuals with replacement)
- An extension: Resample both the residuals and the factor returns for RMRF, SMB, HML, and PR1YR
  - Distribution of "alphas" is sensitive to this "case resampling" as well

- Step 3: Generate the bootstrapped fund returns, assuming no performance for t=1,…,T, for bootstrap sample "b":

$R_i^b - R_f = 0 + \beta*RMRF + s*SMB + h*HML + p*PR1YR + \varepsilon^b$

- <u>Step 4:</u> Estimate the alpha that results simply from sampling variation by:

  $R_i^b - R_f = \alpha^b + \beta^* RMRF + s^* SMB + h^* HML + p^* PR1YR + \varepsilon^b$

- This 4-step procedure is repeated "B" times for each fund for the Carhart measure
- The maximum α^b across all funds is saved for each bootstrap iteration, as well as the maximum t-stat for α^b

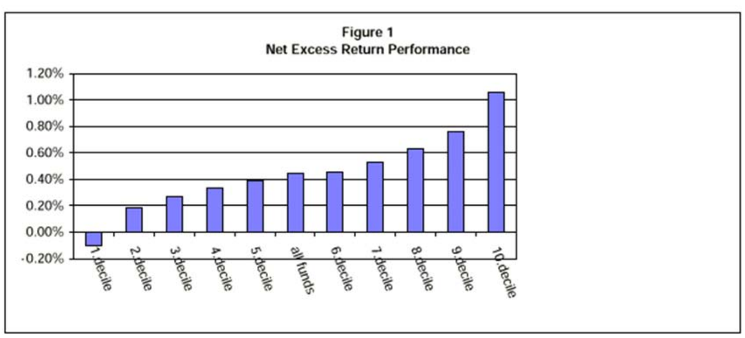
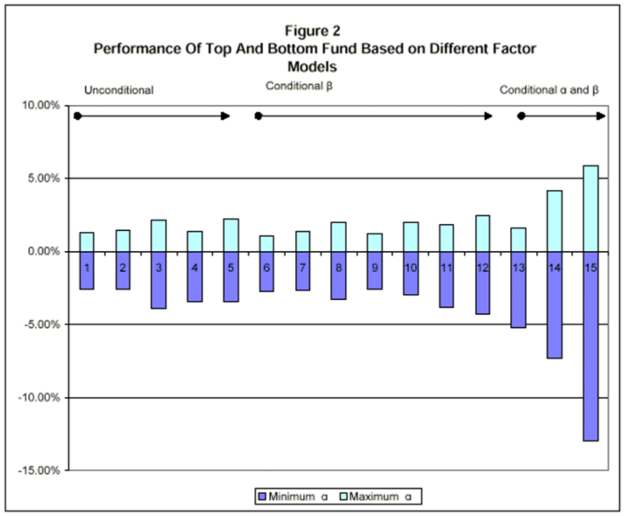


Figure 1. This figure presents fund performance as measured by the funds' average net excess return. For each decile mean performance is reported in percent per month using the full sample of 1783 funds.



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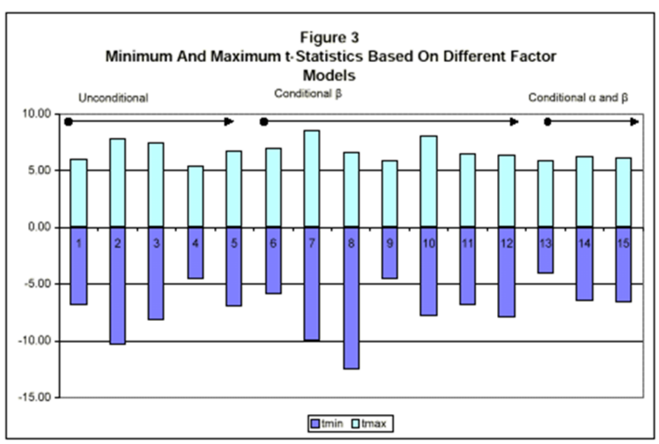

Figure 3. For each factor model this figure shows the t-statistic using heteroskedasticity and autocorrelation consistent standard errors. All models are estimated for funds that have at least 60 observations.

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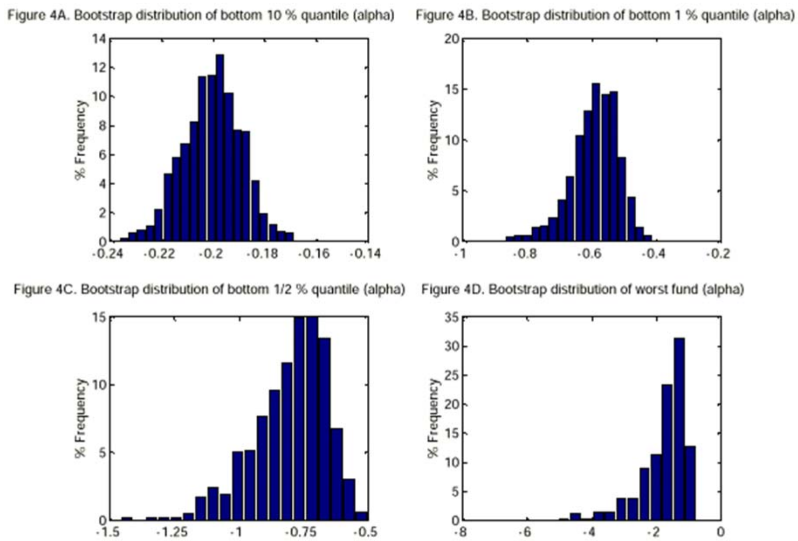


Figure 4. This figure plots the distribution of the estimated alpha for the funds ranked in the bottom 10%, 1%, 0.5% quantile and the worst fund generated by the 1,000 bootstraps.

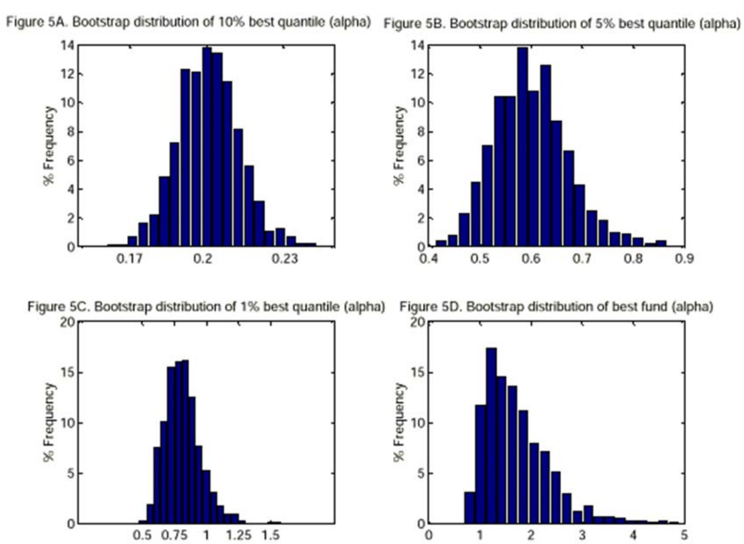
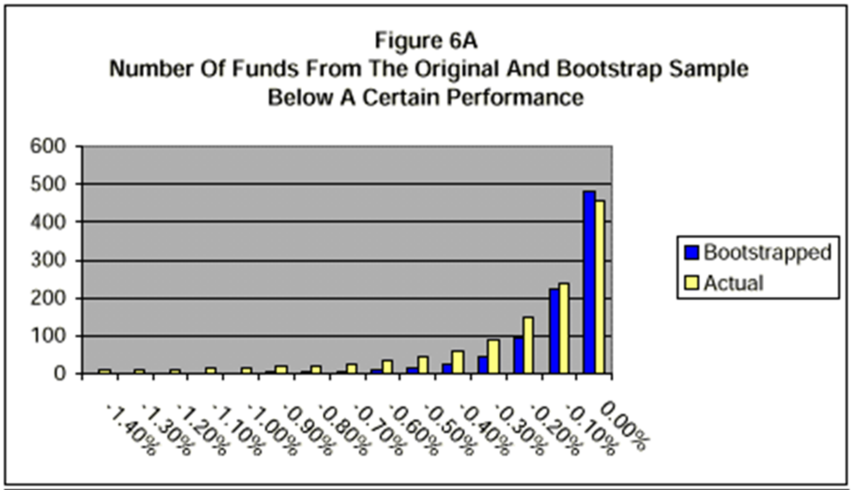


Figure 5. This figure plots the distribution of the estimated alpha for the funds ranked in the top 10%, 1%, 0.5% quantile and the worst fund generated by the 1,000 bootstraps.



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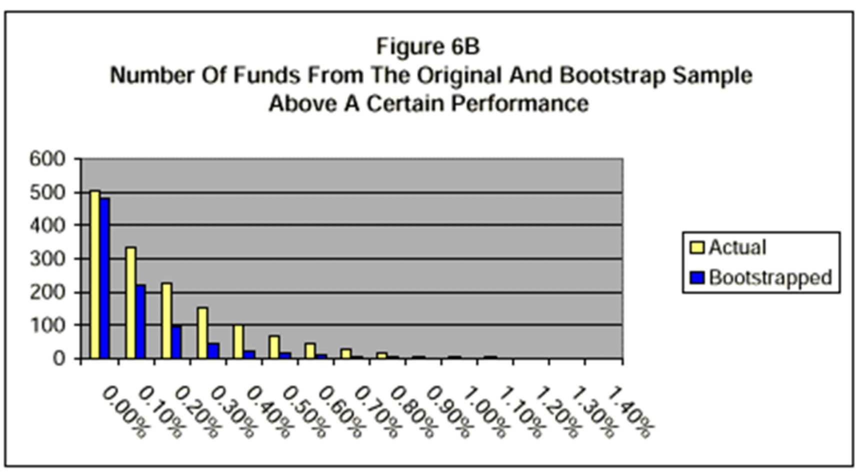

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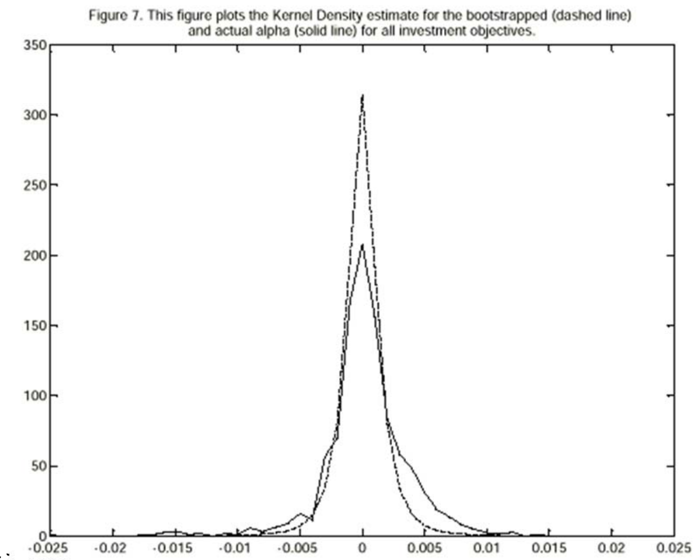

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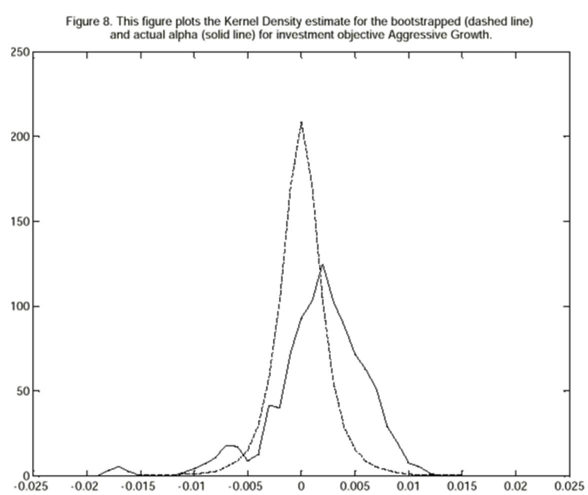

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Figure 9. This figure plots the Kernel Density estimate for the bootstrapped (dashed line) and actual alpha (solid line) for investment objective Growth.

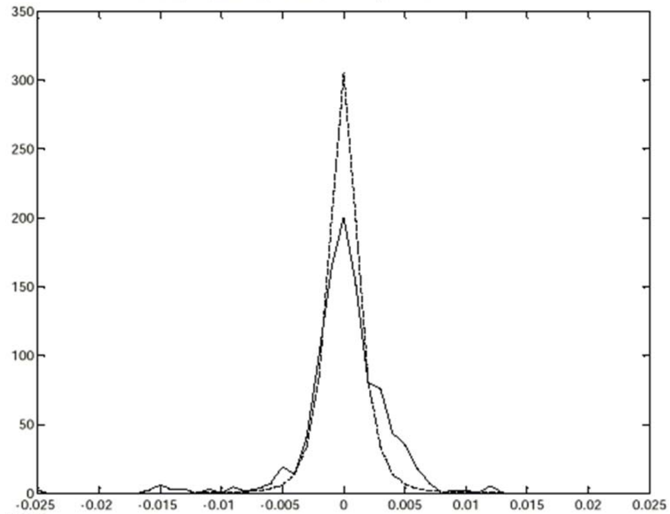
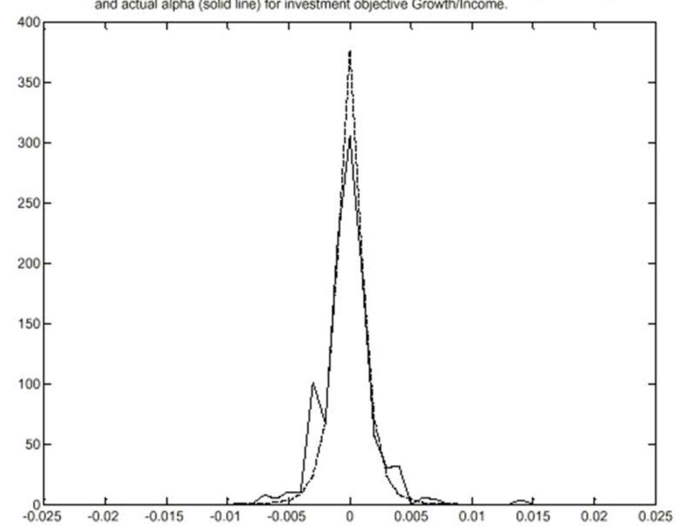
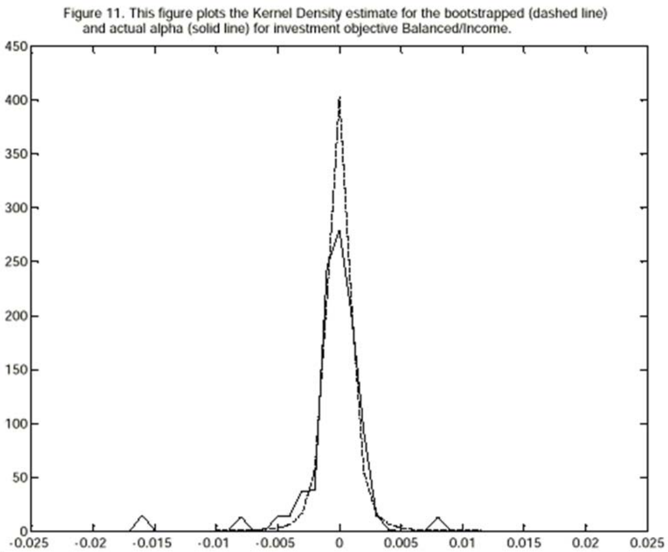


Figure 10. This figure plots the Kernel Density estimate for the bootstrapped (dashed line) and actual alpha (solid line) for investment objective Growth/Income.





Summary

- New bootstrapping technique empirically determines distribution of performance “alphas”
- “Alphas” are non-normal
- Evidence of superior managers—alphas that are higher than we would expect from sampling variation
- Also, evidence of inferior managers—alphas that are lower than we would expect from sampling variation